DIOPHANTINE ANALYSIS.

70. Proposed by CHARLES CARROLL CROSS, Libertytown, Md.

Give methods for decomposing numbers into squares, cubes, or biquadates and show that 61×200^3 is the sum of ten cube numbers and that 844933 is the sum of eleven biquadrates in thirteen different ways. [From *The Mathematical Magazine*, Vol. II, No. 10.]

Solution by the PROPOSER.

Let s_1=sum of products taking 1 number at a time ....(1),
s_2=sum of products taking 2 numbers at a time................(2),
s_3=sum of products taking 3 numbers at a time................(3),
....
s_n=sum of products taking n numbers at a time................(n).

Then $a+b+c+d \ldots\ldots n=s_1$ ....(b_1),

(1)(b_1)$-$2(2) gives

$a^2+b^2+c^2+d^2 \ldots\ldots n^2=s_1^2-2s_2$ ....(b_2),

(1)(b_2)$-$[(2)(b_1)$-$3(3)] gives

$a^3+b^3+c^3+d^3 \ldots\ldots n^3=s_1^3-3s_1s_2+3s_3$ ....(b_3),

(1)(b_3)$-$[(2)(b_2)$-$(3)(b_1)$-$4(4)] gives

$a^4+b^4+c^4+d^4 \ldots\ldots n^4=s_1^4-4s_1^2s_2+4s_1s_3+2s_2^2+4s_4$ ....(b_4),

(1)(b_4)$-$[(2)(b_3)$-$(3)(b_2)$-$(4)(b_1)$-$5(5)] gives

$a^5+b^5+c^5+d^5 \ldots\ldots n^5=s_1^5-5s_1^3s_2+5s_1^2s_3+5s_1s_2^2+5s_1s_4-5s_2s_3+5s_5$ ..(b_5),

etc. etc. etc.

[In (b_5) let $s_1=x$, $s_2=-xy$, $s_3=2xy^2$, $s_4=y^2(x^2-y^2)$, and $s_5=\frac{1}{5}y^5$, then $a^5+b^5+c^5+d^5+e^5+f^5=(x+y)^5$. This answers Dr. Drummond's note on page 182, Vol. V., of the MONTHLY.]

71. Proposed by A. H. BELL, Hillsboro, Ill.

Find five numbers such that the product of any two plus 1 will equal a square.

I. Partial Solution by CHARLES C. CROSS, Libertytown, Md.

We readily find four numbers to be $(n-1)$, $(n+1)$, $4n$, and $4n(4n^2-1)$.

Let n=2, 3, 4, and so on, and we get

1,	3,	8,	120,	also	1,	8,	15,	528,
2,	4,	12,	420,		2,	12,	24,	2380,
3,	5,	16,	1008,		3,	16,	33,	6440,
4,	6,	20,	1980,		4,	20,	42,	13572,
	etc., etc.					etc., etc.		

From which we get a more general formula for four numbers and find it to be, m, $n^2-1+(m-1)(n-1)^2$, $n(mn+2)$, $m(m^2n-2mn+mn^2+4n-2)^2+(m^2n-2mn+mn^2+4n-2)$.

If there can be found a value for n that will render $(n-1)(128n^3-8n-8)$ $(256n^6-256n^4+63n^2-2n-1)+(256n^6-288n^4-32n^3+65n^2+2n+1)^2=\square$ then the five numbers are $(n-1)$, $(n+1)$, $4n$, $4n(4n^2-1)$, and $[(256n^6-256n^4+63n^2-2n-1)(128n^3-8n-8)]/(256n^6-288n^4-32n^3+65n^2+2n+1)^2$; but life is too short to attempt this.

Dr. Zerr says, "I have not been able to find the five numbers."

If you get no answers to problem 71, I can give Legendre's (improved), and it is a fractional answer for the fifth number. Mr. Wilkes has found four numbers, as follows: $n-1$, $n+1$, $4n$, $16n^3-4n$. Taking these for four of them then I can prove that if there is another integral number it must end in 0.

I have investigated, up to numbers having over five thousand digits, without finding any to answer. A. H. BELL.

I have given considerable time to problem 71, but as yet have failed to obtain five numbers. I have found general values for three such numbers, viz., m, $n(mn\pm2)$, and $(n+1)[(n+1)m\pm2]$. I have also found partial general values for four such numbers, viz., m, $m\pm2$, $4(m\pm1)$, and $16(m\pm1)^3-4(m\pm1)$; but one of the sets of four numbers that I obtained by inspection, can not be obtained by this formula. The set is 1, 8, 15, 528. M. A. GRUBER.

MISCELLANEOUS.

63. Proposed by F. P. MATZ, D. Sc., Ph. D., Professor of Mathematics and Astronomy, Irving College, Mechanicsburg, Pa.

Show that the path of a projectile moving with a constant velocity is an inverted catenary of equal strength.

Solution by GEORGE R. DEAN, Professor of Mathematics, University of Missouri School of Mines and Metallurgy, Rolla, Mo.

The differential equations are, obviously,

$$\frac{d^2y}{dt^2}=-g, \quad \frac{ds}{dt}=c.$$

Whence, $\dfrac{dy}{dt}=-gt+k$, $\dfrac{dx}{dt}=\sqrt{c^2-(gt-k)^2}$, $x+k'=-\dfrac{1}{g}\cos^{-1}\dfrac{gt-k}{c}$.

By division, $$\frac{dy}{dx}=\frac{-\cos g(x+k')}{\sqrt{c^2-c^2\cos^2 g(x+k')}}=-\cot g(x+k'),$$

$$\text{or, } \cot^{-1}\frac{dy}{dx}=-g(x+k').$$

Differentiation gives